PH.D. SYNOPSIS/WORK PLAN

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A Study of Existence and Behaviours of Random Solution for Random Differential Equations

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Review of the Literature

A quantitative description of a natural phenomenon is called a mathematical model of that phenomenon. The mathematical models have been used to describe a particular phenomenon or processes of the universe. The attempt to solve physical sciences problems led gradually to mathematical models involving an equation in which a function and its derivatives play important roles. However, the theoretical development of Differential Equations has its origins rooted in a small number of mathematical problems. These problems and their solutions led to an independent discipline with the solution of such equations an end in itself. According to some historians of mathematics, the study of differential equations began in 1675, when Gottfried Wilhelm von Leibniz (1646-1716), Isaac Newton (1642-1727)1682, Leibniz, James Bernoulli (1654-1705] Leonhard Euler (1707-1783) Alexis Claude Clairaut (1713-1765), Joseph Louis Lagrange (1736-1813). In 1696. John Bernoulli, a student, rival, and equal of his older brother James, gave a main impetus to the study of differential equations through posing his famous brachistochrone problem of finding the equation of the path down which a particle will fall from one point to another in the shortest time.

Origin of the Problem

Deterministic models typically written in terms of systems of ordinary differential equations have been very successfully applied to an endless variety of physical phenomena for example Newton's law of motion, however, there are

situations where deterministic laws alone are inadequate, particularly as the number of interacting species becomes small and the individual motion of microscopic bodies has observable consequences.

The interactions that are eliminated from macroscopic models make themselves felt in other ways a ball of crumpled paper experiences a strong drag force in flight due to air-resistance, a constitutive equation must be provided to fix the visco-elastic properties of a fluid etc. Drag forces, viscosity, electrical resistance, these are all slices of the microscopic dynamics left behind in the macroscopic models when the enormous degrees of freedom in the original many body problem were integrated away to leave a simple deterministic description. These shadows of microscopic motion are called fluctuations or noise.

The most famous example of fluctuations in a physical system is Brownian motion. Using a microscope, Robert Brown observed and documented the motion of large pollen grains suspended in water. The pollen grains incessantly in erratic and irregular motion. Brown tried different materials and different solvents, and still the motion of these particles continued. This was a time when most scientists did not believe in atoms or molecules so the mechanism responsible remained a mystery for nearly century. After that, the study of Brownian motion successfully described by Einstein in 1905, Smolouchowski in 1906, Langevin in 1908, Ornstein and Uhlenbeck in 1930 etc. The nondeterministic nature of phenomena in the general areas of biological and Physical Sciences that the mathematical models of such phenomena frequently result in random equations.

A large class of physical sciences important problems is described by ordinary differential equations. In taking various random effects in to account, it is naturally that ordinary differential equations involving random elements that is random ordinary differential equations. The random ordinary differential equations will play an increasingly prominent role in natural and physical sciences. The theory of random equations is still in the formative stage; however, there is a very extensive literature dealing with four basic classes of random

equations, namely: (1) random algebraic equations, (2) random difference equations, (3) random differential equations, and (4) random integral equations. Other random equations can be considered, for example, random differential-difference equations, random integro-differential equations, and rather general random functional equations. However, for the purposes of this exposition we regard the above four classes as the basic classes of random equations. We present a list of survey articles and books which deal with the basic classes of random equations and their applications are as in Random algebraic equations by Bharucha-Reid ,Random difference equations by Aoki , Astrom, Bharucha-Reid, and Jazwinski, Random differential equations: Arnold , Astrom, Bucy and Joseph , Gihman and Skorohod, Jazwinski , Khasminskii , Kushner, Saaty , Srinivasan and Vasudevan , Stratonovich, Wong , and Wonham ,and Random integral equations by Dyrikin, Gihman and Skorohod , Ito and McKean , McKean, Skorohod, and Tsokos and Padgett. Also Random processes by Applied Stochastic Processes in science and engineering by M. Scott 2013, An Introduction to Stochastic Modeling by Howard M.Taylor, Samuel Karlin, Academic Press, USA, 1998.

As in the theory of differential equations of ordinary nonrandom functions, the development of random differential equations deals with in some sense random, the existence and uniqueness of the solution process and the study of certain characterization of solutions such as weak solution, strong solution, stability of solutions, asymptoticness and attractivity results of solutions. In recent years, random approach has achieved a high level of maturity and its circle of influence contains physics, chemistry, biology economics management, sociology, psychology, medicine and all engineering disciplines. Because of our increasingly stringent demands on quality and accuracy in dealing with increasingly complex systems, applied workers have become aware of the

importance of random function theory as a weapon of rational approach toward a wide variety of problems.

The study of random differential equations is a wide field in pure and applied mathematics, physical sciences, engineering, biology, medicine and economics. All of these disciplines are concerned with random differential equations of various types. Pure mathematics focuses on the existence, uniqueness and various characterizations of random solutions. Applied mathematics gives the rigorous justification of approximating solutions.

Status: International

Mathematical modeling of random process, random differential equations and study of its solution is completely new topic in the research area. Mathematical modeling of random process is studied in an international level long time by Robert Brown, Einstein in 1905, Smolouchowski in 1906, Langevin in 1908, Ornstein and Uhlenbeck in 1930 and till etc. by Einstein in 1905, Smolouchowski in 1906, Langevin in 1908, Ornstein and Uhlenbeck in 1930 etc. Recently many researcher work on random process of all fields natural and physical sciences in universe. Study of random differential equations under existence, uniqueness of random solution and various characterizations of random solution such as weak solution, strong solution, asymptotic behavior, stability and attractivity results etc. rare work doing in an international level. Some of researchers work on above are as Aoki, M., Astrom, K. J., Bharucha-Reid, A. T., K. Deimling, Omid S. Fard, Granas and J. Dugundji, Gihman, I. I., and Skorohod, A. V., Izdat. Naukova Dumka, Kiev, S. Heikkila and V. Lakshmikantham, J. Heimmelberg, ,S. Itoh, Jazwinsjd, Khasminskii, R. Z., Kushner, H. J., Ha. Kushner, H. J., Skorohod, A. V.,Tsokos, C. P., and Padgett, W. J., Wonham, W, M., T.T. Soong

It is clear from the above list of references that random differential equations have been the subject of rather extensive research. This, however,

should not be surprising; for deterministic differential equations are utilized as models for a wide variety of phenomena considered in the applied mathematical sciences and, consequently, play a dominant role in applied mathematics.

Status: National

Mathematical modeling of random process is not detail studied in national level concerned to physical sciences problems but engineering fields are studied by some sense. The study of random differential equations and its solution is studied by some researchers are as Srinivasan, S. K., and Vasudevan, R., B.C. Dhage and others. This work will be a very good contribution in the area of random differential equations which is useful for scientist, engineers, ocea nographers and all physical sciences problems.

Objective of the Research Work

The random differential equations includes the classical differential equations as a special case. The study of random differential equations is more than the study of classical differential equations and it is applicable in classical, random, natural or physical phenomena of universe. The present work highlights of the following:

1. To study of nonlinear boundary value problems of random differential equations involving discontinuities under existence, uniqueness and some characterizations of random solutions.
2. To study nonlinear boundary value problems or periodic boundary value problems of random differential equations under existence of positive random solutions and characterizations of random solutions.
3. To study perturbed nonlinear boundary value problems or periodic boundary value problems of random differential equations under existence of positive, random solutions and its characterizations.
4. Applications.

Methodology

There are mainly three approaches for dealing with the nonlinear random or stochastic differential equations via,

I) Approximation Method,
II) Measurable Selection Method, and
III) Random Fixed Point Method.

In the first approach of approximation method, for given fixed values of the random parameters, the deterministic or classical solution is obtained by using the available or classical suitable method and then the measurability of the classical solution with respect to these parameters is proved step by step successively which thereby gives the random solutions. This approachd has been exdploited in the works of Ladde and Lakhsmikantham.

The second approach of measurable selection method is very interesting, but it has a limitation that it is necessary to known in advance that the random differential equation in question has a deterministic solution.Then, the measurability of this solution with respect to the random parameter is proved by applications of some known results from probabilistic functional analysis. See N.S. Papageorgiou.

The third approach of random fixed point method is the most powerful one which on application directly yields the existence of random solutions. The work in this direction may be found in Bharucha- Reid in Random Integral Equations, Academic press, New York (1972). We followed the approach Itoh in Random fixed point theorems with application to Random Differential Equations in Banch spaces, J. Math. Anal. Appl. 67 (1979), 261-273 in the work of this research topc. Hence, we choose random fixed point method throughout the thesis to prove existence, existence of extremal solutions and other results.

Importance

1. The study of random differential equations has a rather recent origin. Today, it has become a subject of intensive research. The reason for this vigorous research activity is clear if we consider the problem of mathematical modeling of practically all physical phenomena in nature. The translation from a

physical problem to a set of mathematical equation is never perfect. This is due to a combination of uncertainities, complexities and ignorance on our part which inevitably cloud our mathematical modeling process. In order to take them in to account, mathematical equations with random coefficient offer a natural and rational approach. For example wave propagations in inhomogeneous media, systems and structures with parametric excitations and dynamics of imperfectly known systems in physics, engineering, biology, medicine and economics.

2. In recent years, random approach has achieved a high level of maturity and its circle of influence contains physics, chemistry, biology economics management, sociology, psychology, medicine and all engineering disciplines. Because of our increasingly stringent demands on quality and accuracy in dealing with increasingly complex systems, applied workers have become aware of the importance of random function theory as a weapon of rational approach toward a wide variety of problems.
3. The study of random differential equations is a wide field in pure and applied mathematics, physical sciences, engineering, biology, medicine and economics. All of these disciplines are concerned with random differential equations of various types. Pure mathematics focuses on the existence, uniqueness and various characterizations of random solutions. Applied mathematics gives the rigorous justification of approximating solutions. This work will be a very good contribution in the area of random differential equations.

Year – Wise Plan of Work

First Year

Survey and collection of basic materials required for the topic. This includes collection of journals, Books and research papers related to the topic. Visit for few libraries of well known institutions like TIFR, IIAC, IITs, National Science Academy, National Institute of Mathematics and meet, discuss with faculty members towards for the direction of the topic.

Second Year

The collected material will be processed, analyzed on direction of the research topic and attempt will be made to create or discover the findings in the direction of research topic.

Third Year

The outcome result will be presented in national and international gathering such as seminars, workshops, conferences etc. during this year. Also publishing the outcome results in national, international journals during this year. Writing and submission of research work to Swami Ramanand Teerth Marathwada University, Nanded.

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।। सा विद्या या विमुक्तये ।।

स्वामी रामानंद तीर्थ मराठवाडा विद्यापीठ, नांदेड

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Certificate of Allocation

This is to certify that meeting of Research Allocation Committee for **PET-2020** in the Subject of **Mathematics** Under the **Faculty of Science & Technology** is Held on **12th July, 2021.**

In the said meeting Mr./~~Ms.~~ Jagtap Nitin Maruti has been Allocated to Supervisor, Dr. B. S. Palimkar

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to complete his/her Ph.D. work.

Dean
Faculty of Science & Technology
And Chairman, RAC

Copy to.

1. Supervisor, with request to accommodate and supervise the aforesaid Research Scholar.
2. Research Centre for information and necessary action.
3. Research scholar, with instructions to contact the Supervisor for preparing the Synopsis is to be submitted to P.G. Section of the university within two weeks, which will be placed before Research & Recognition Committee for its consideration. The Research scholar has to appear before the RRC at the time it's meeting. Which will be notified on university website in due course of time. After Approval of research topic by RRC researcher has to submit soft copy of synopsis in C.D. at central library of S.R.T.M. University.

टिप: -

१. दि. १२/०५/२०१५ च्या मा. विद्या परिषद बैठकीत विषय क्र. ०२/३४-२०१५ ब नुसार आर.ए.सी. ने गाईडकडे विद्यार्थी दिल्यानंतर गाईडने नकार दिल्यास त्यांना पुढील दोन वर्षे विद्यार्थी देण्यात येऊ नयेत, असे ठरलेले आहे.

२. दिनांक ०६/०३/२०१९ च्या मा. अधिष्ठाता मंडळाच्या बैठकीत विषय क्र.१९/१२-२०१९ नुसार आर.ए.सी. झालेल्या तारखेनंतर एक वर्षाच्या आत संशोधकाने आर.आर.सी. बैठकीस उपस्थित राहून संशोधन शीर्षक मान्य करणे अनिवार्य राहील. तसेच आर.आर.सी. बैठकीत संशोधन शीर्षक मान्य झाल्यानंतर एक वर्षात विहित शुल्क भरून संशोधन केंद्र तसेच पदव्युत्तर विभाग येथे रूजू होणे बंधनकारक आहे. आर.ए.सी. नंतर वर्षभरात दोन आर.आर.सी. झाल्या नाहीत तर दोन आर.आर.सी. होईपर्यंत मुदत ग्राह्य धरली जाईल. आर.ए.सी. नंतर आर.आर.सी. समोर वेळेत उपस्थित राहून देखील आर.आर.सी. ने विषयात काही सुधारणा सुचविली असेल तर अशा उमेदवारांना पुढील एक वर्षाच्या आत विषय मान्यता घेऊन त्याच वर्षी संशोधन केंद्रावर रूजू होणे अनिवार्य राहील. तसे केल नाहीतर ती नोंदणी ग्राह्य धरली जाणार नाही.

३. पीएच.डी.चा संशोधन कालावधी सहा वर्षे असेल. या कालावधीनंतर मुदतवाढ दिली जाणार नाही.

४. दि.२०/०६/२०२० च्या मा.विद्या परीषदेच्या निर्णयानुसार संबंधित उमेदवारांनी Research Methodology, Computer Application, Research and publication Ethics या तीन विषयाची कोर्सवर्क परीक्षा उत्तीर्ण झाल्याशिवाय त्यांची पीएच.डी. नोंदणी कायम केली जाणार नाही.